



EE 232 Lightwave Devices

Lecture 16: p-i-n Photodiodes and Photoconductors

Reading: Chuang, Chap. 15 (2nd Ed)

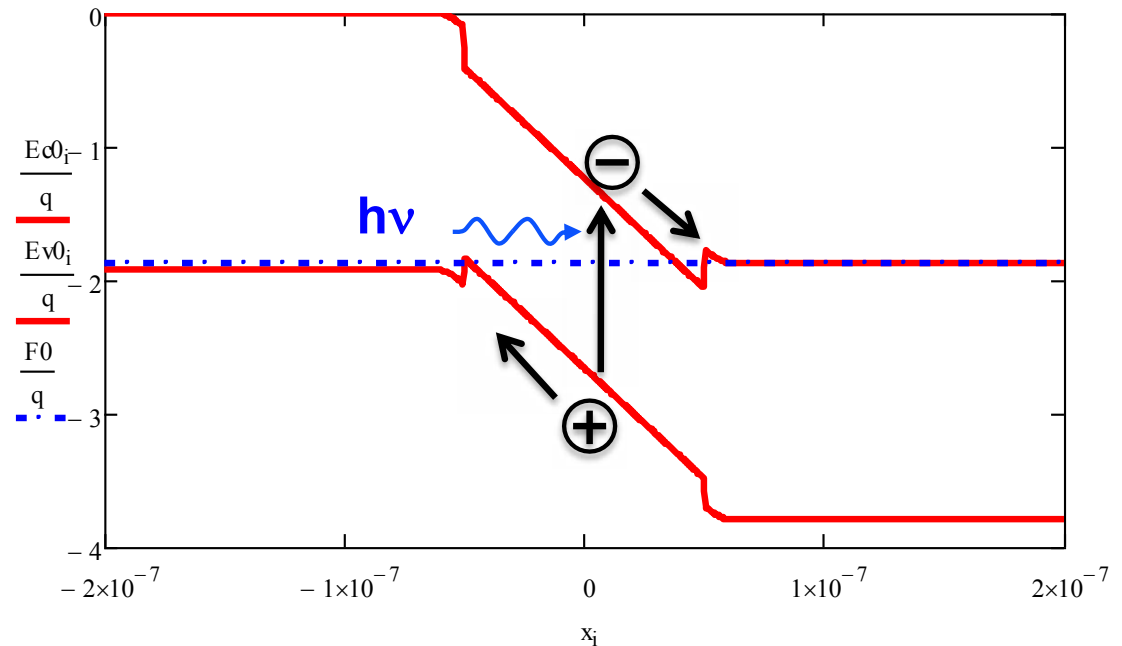
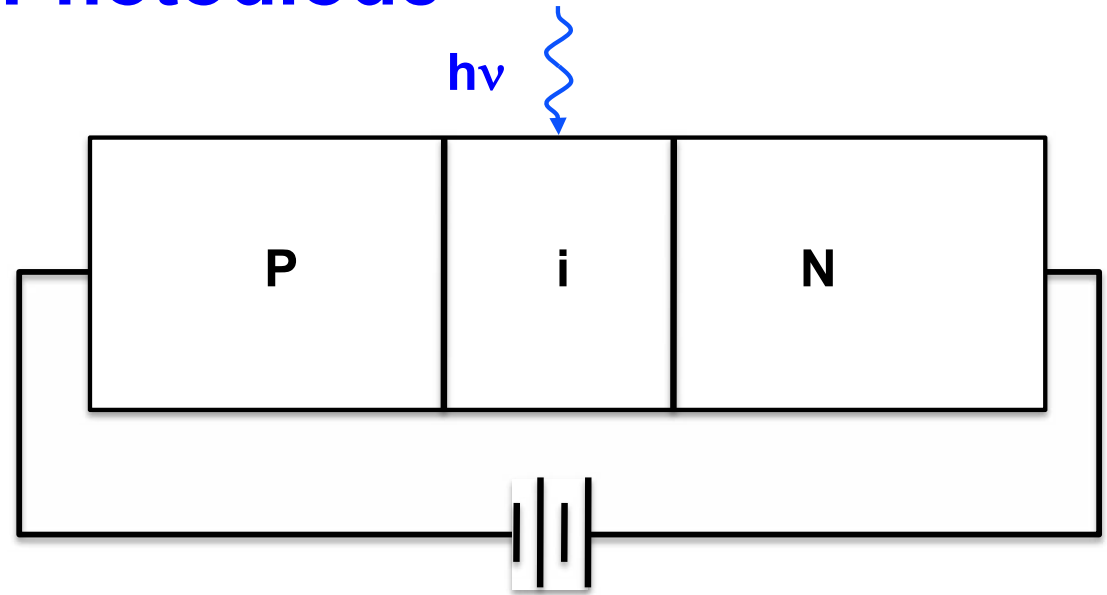
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P-i-n Photodiode

- Reverse-biased p-i-n junction
- Most of the voltage drop across the i-region, the main absorption region
- High field separates photogenerated electron and hole
- Large bandgap materials are used for P and N if possible
- Fast response
- Low noise
- No gain (quantum efficiency < 100%)





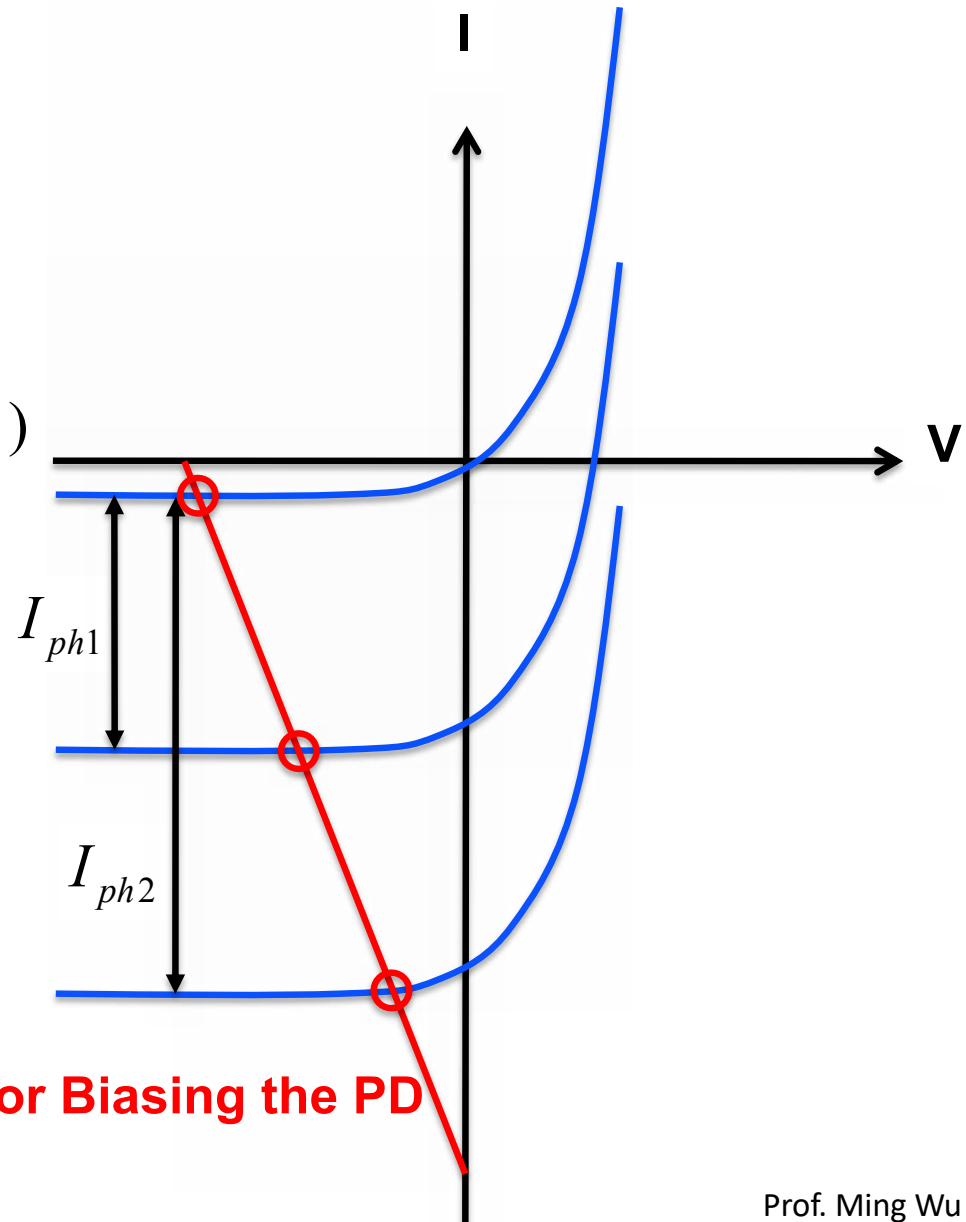
I-V Curve

Dark current: $I = I_0(e^{\frac{qV}{k_B T}} - 1)$

Photocurrent: $I_{ph} = \frac{\eta q}{\omega} P_{opt}$

Quantum efficiency: $\eta = \eta_i(1 - R)(1 - e^{-\alpha d})$

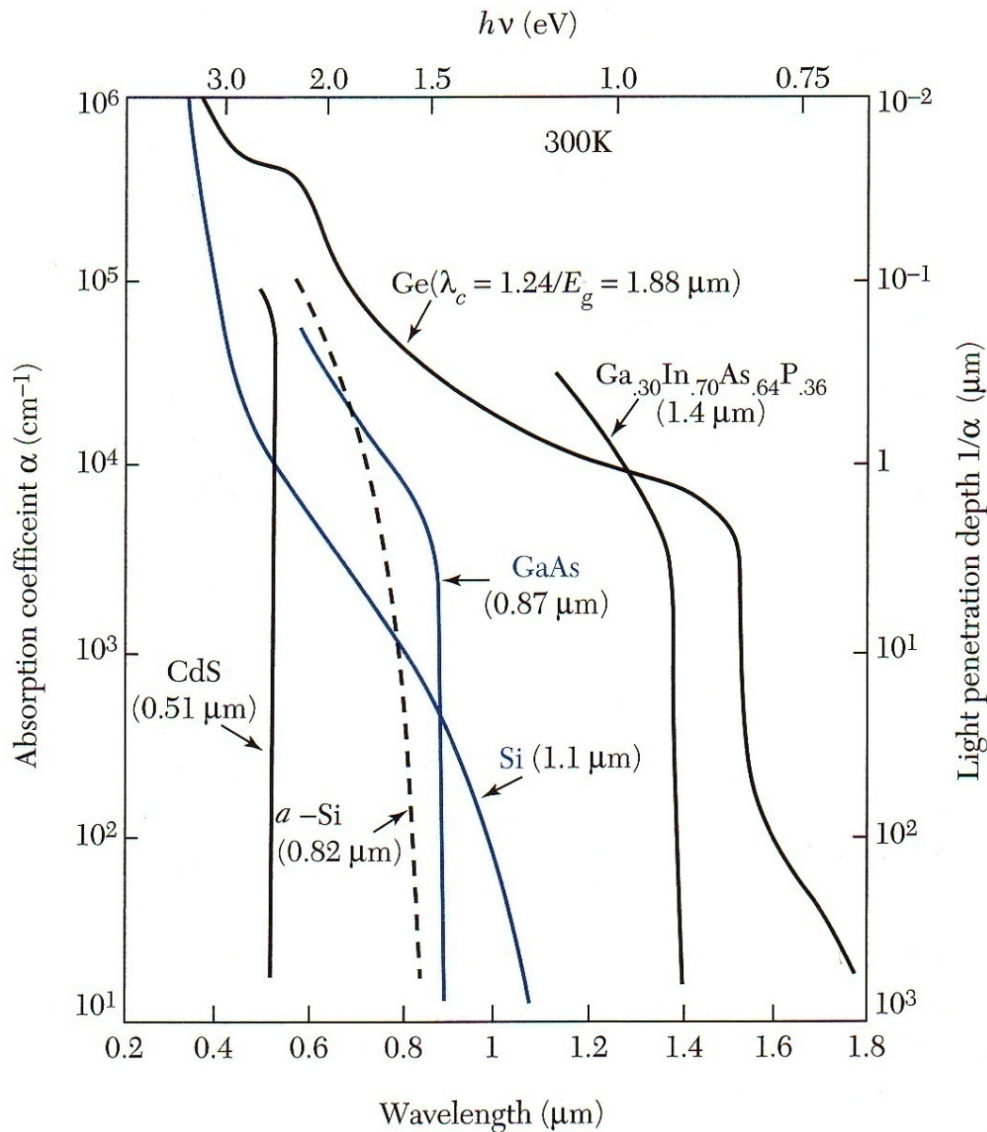
Total current: $I = I_0(e^{\frac{qV}{k_B T}} - 1) + I_{ph}$



Load Line for Biasing the PD



Absorption Coefficient



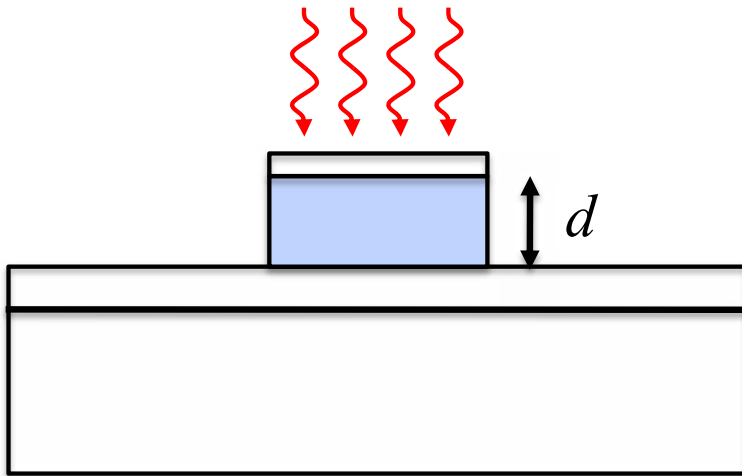
- Light intensity decays exponentially in semiconductor:

$$I(x) = I_0 e^{-\alpha x}$$

- Direct bandgap semiconductor has a sharp absorption edge
- Si absorbs photons with $h\nu > E_g = 1.1 \text{ eV}$, but the absorption coefficient is small
 - Sufficient for CCD
- At higher energy ($\sim 3 \text{ eV}$), absorption coefficient of Si becomes large again, due to direct bandgap transition to higher CB



Two Types of p-i-n Photodiodes



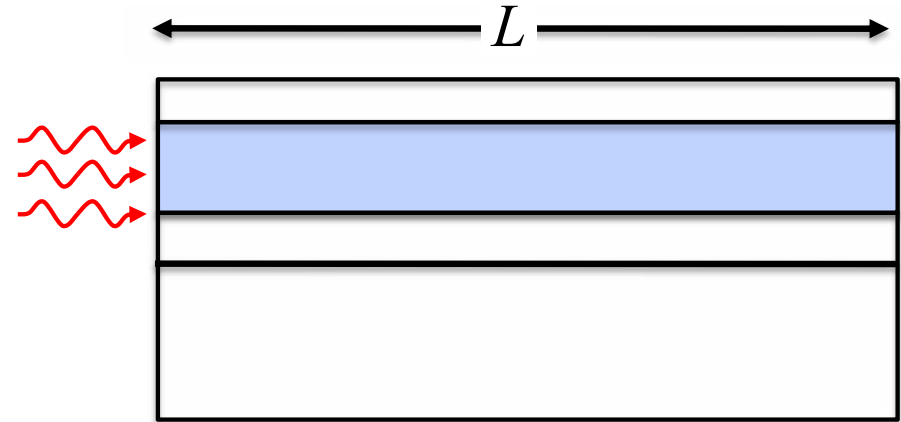
Surface-Illuminated p-i-n

$$\eta = \eta_i(1 - R)(1 - e^{-\alpha d})$$

η_i : internal quantum efficiency

R : reflectivity

d : absorption layer thickness



Waveguide p-i-n

$$\eta = \eta_i(1 - R)(1 - e^{-\Gamma\alpha L})$$

η_i : internal quantum efficiency

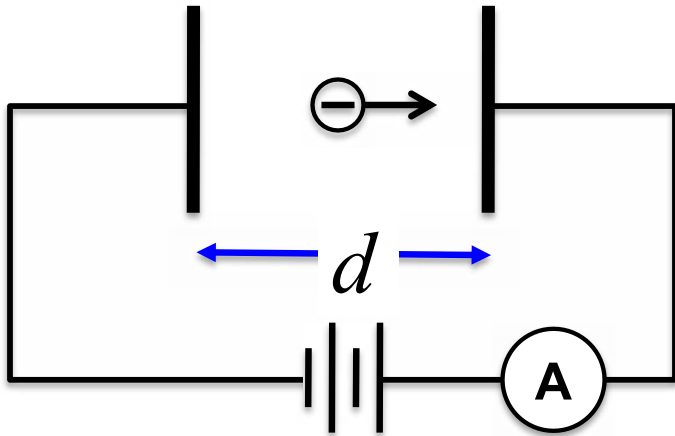
R : reflectivity

Γ : confinement factor

L : length of waveguide PD



Ramo's Theorem



The current caused in external circuit by a moving charge q moving at a velocity $v(t)$ in a parallel plate with a separation of d and a voltage bias of V is

$$i(t) = \frac{qv(t)}{d}$$

Proof:

Work done on the charge:

$$W = \text{Force} \times \text{Displacement}$$

$$= qEdx = q \frac{V}{d} dx$$

Work provided by power supply:

$$W = i(t)Vdt$$

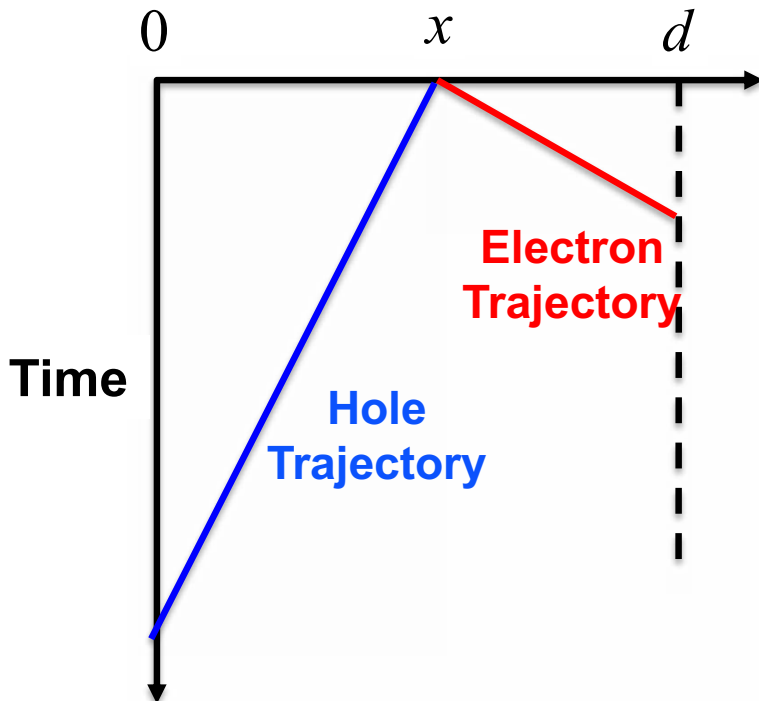
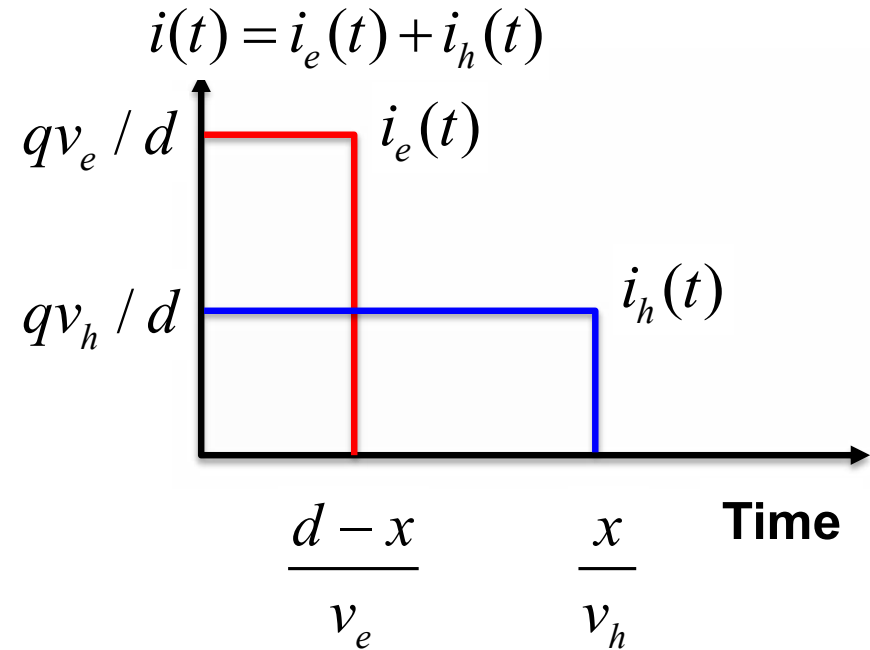
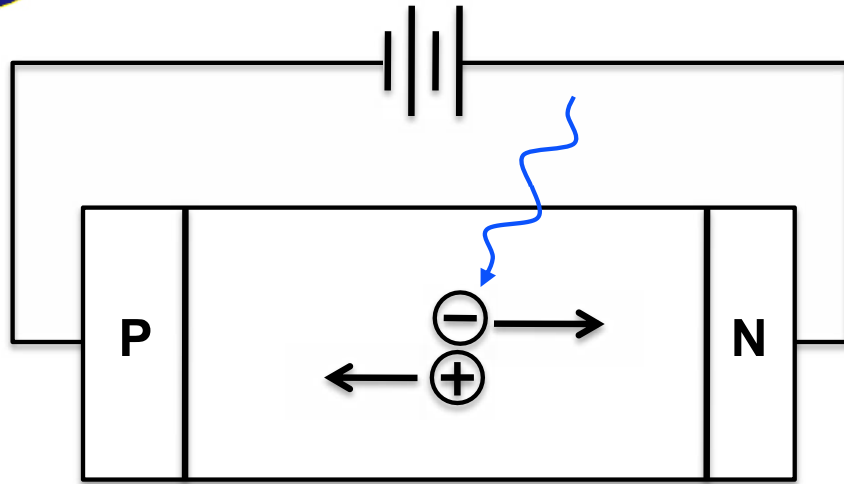
\Rightarrow

$$i(t)Vdt = q \frac{V}{d} dx$$

$$i(t) = \frac{q}{d} \frac{dx}{dt} = \frac{qv(t)}{d}$$



Response of One Photogenerated Electron-Hole Pair



Total charge generated:

$$Q = \int_0^{\infty} i_e(t) dt + \int_0^{\infty} i_h(t) dt$$

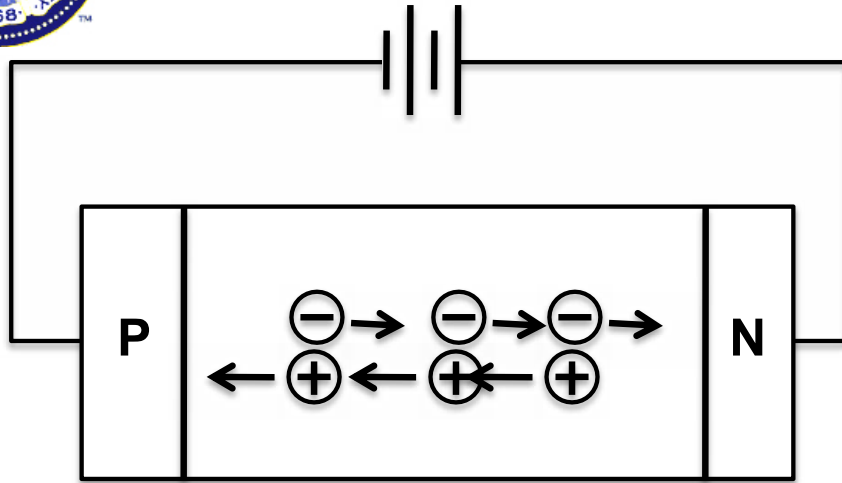
$$= \frac{qv_e}{d} \frac{d-x}{v_e} + \frac{qv_h}{d} \frac{x}{v_h} = q$$

One absorbed photon \rightarrow

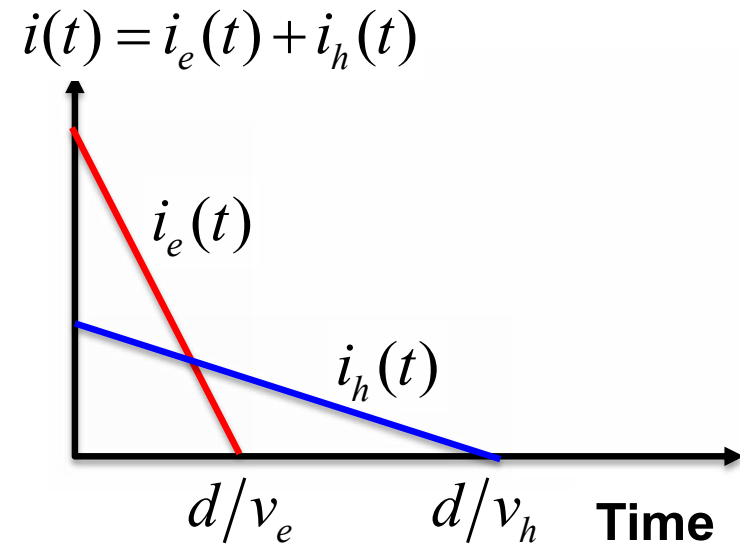
one charge detected Prof. Ming Wu



Transit Time



Many electron-hole pairs generated



Electron current ends when the last electron generated near P-side reaches N-electrode: $t = d/v_e$

Hole current ends when the last hole generated near N-side reaches P-electrode: $t = d/v_h$

Hole is usually slower \rightarrow A conservative estimate of the transit time:

$$\tau_t = \frac{d}{v_h}$$



Total Response Time of p-i-n

(1) RC time:

$$\tau_{RC} = RC = R \frac{\epsilon A}{d} \quad (A: \text{ area of p-i-n})$$

(2) Transit time:

$$\tau_t = \frac{d}{v_h}$$

Total response time:

$$\tau = \tau_{RC} + \tau_t$$

$$f_{3dB} \approx \frac{1}{2\pi\tau}$$

Absorption layer thickness

for optimum frequency response:

$$\tau = \tau_{RC} + \tau_t = \frac{R\epsilon A}{d} + \frac{d}{v_h}$$

$$\tau \geq 2 \sqrt{\left(\frac{R\epsilon A}{d}\right) \left(\frac{d}{v_h}\right)}$$

$$f_{3dB} \approx \frac{1}{2\pi\tau} \leq \frac{1}{4\pi} \sqrt{\frac{v_h}{R\epsilon A}} = f_{3dB, \max}$$

Optimum bandwidth occurs when

$$\frac{R\epsilon A}{d} = \frac{d}{v_h}$$

$$d_{\text{optimum}} = \sqrt{R\epsilon A v_h}$$



More Rigorous Analysis of p-i-n Response Time

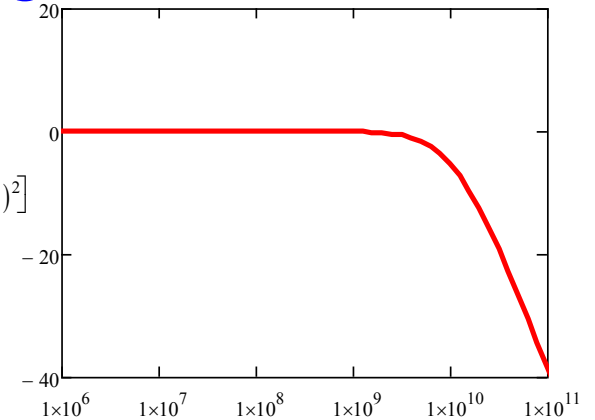
Small-signal analysis: assume the input light is modulated at frequency ω , the photocurrent is proportional to

$$|i(t)| \propto \left| \frac{1}{1 + j\omega RC} \right| \left| \frac{\sin^2\left(\frac{\omega\tau_t}{2}\right)}{\left(\frac{\omega\tau_t}{2}\right)^2} \right|$$

The first term is single-pole response from RC, while the second term is the phase delay due to transit time response.

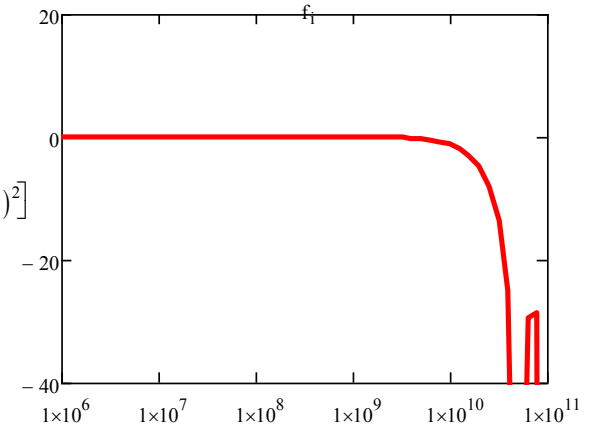
RC

$$20 \cdot \log_{10} [|H1(f_i)|^2]$$



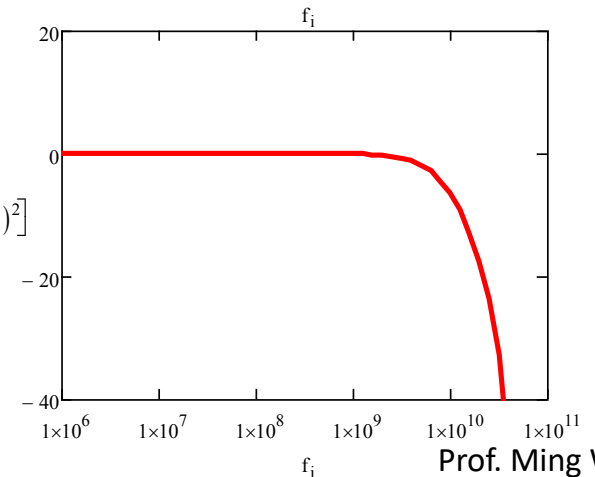
Transit Time

$$20 \cdot \log_{10} [|H2(f_i)|^2]$$



Total

$$20 \cdot \log_{10} [|H3(f_i)|^2]$$





Comparison of Numeric Examples

Example:

$$\tau_{RC} = 14.4 \text{ ps}$$

$$\tau_t = 20 \text{ ps}$$

$$f_{3dB} = \frac{1}{2\pi} \frac{1}{\tau_{RC} + \tau_t} = 4.6 \text{ GHz}$$

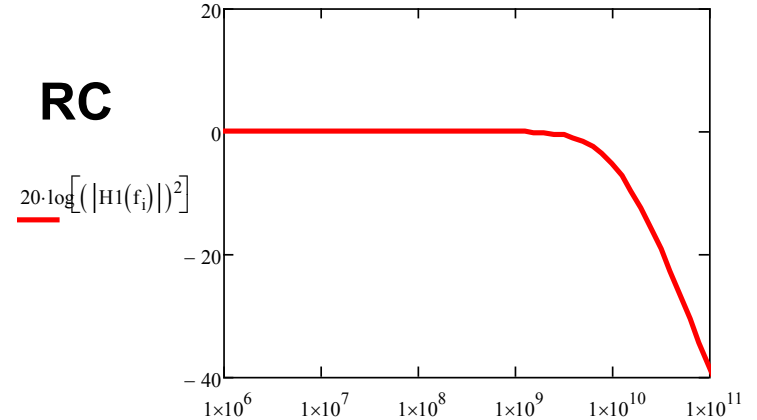
$$|i(t)| \propto \left| \frac{1}{1 + j\omega RC} \right| \left| \frac{\sin^2\left(\frac{\omega\tau_t}{2}\right)}{\left(\frac{\omega\tau_t}{2}\right)^2} \right| = |H(\omega)|$$

Solving $|H(\omega)| = \frac{1}{\sqrt{2}}$, $f_{3dB} = 9.7 \text{ GHz}$

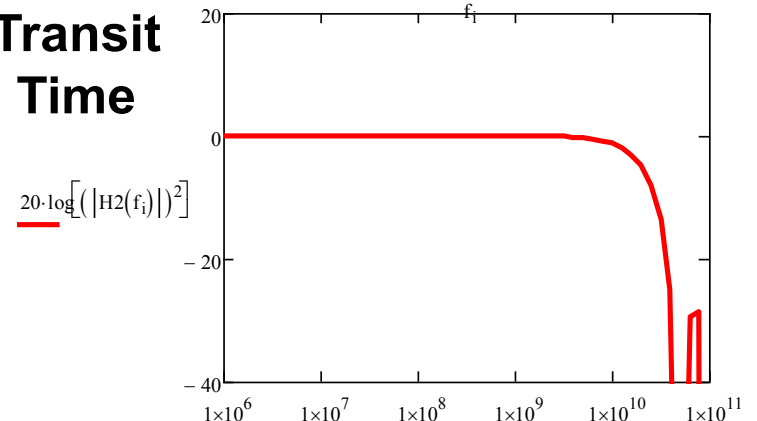
The discrepancy is smaller when RC dominates, and larger when transit time dominates.

(Transit time response has a sharp drop-off).

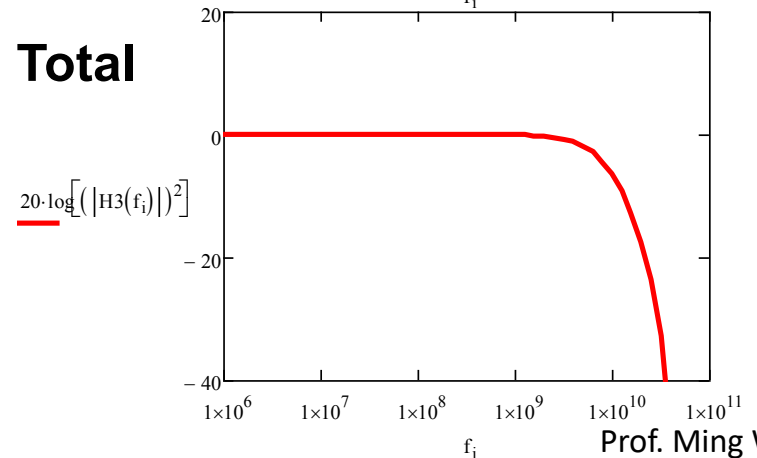
RC



Transit Time



Total





Bandwidth-Efficiency Product

(1) For surface-illuminated p-i-n (assume AR coating: $R=0\%$), in the extreme of thin absorbing layer and transit-time-dominated response:

$$\eta = \eta_i(1 - e^{-\alpha d}) \approx \eta_i(1 - (1 - \alpha d)) = \eta_i \alpha d$$

$$f_{3dB} \approx \frac{1}{2\pi} \frac{v_h}{d}$$

$$\text{Bandwidth-efficiency product: } f_{3dB} \times \eta \approx \left(\frac{1}{2\pi} \frac{v_h}{d} \right) (\eta_i \alpha d) = \frac{\eta_i \alpha v_h}{2\pi}$$

(2) On the other hand, the efficiency of waveguide p-i-n is

$$\eta = \eta_i(1 - e^{-\Gamma \alpha L}) \approx \eta_i \Gamma \alpha L$$

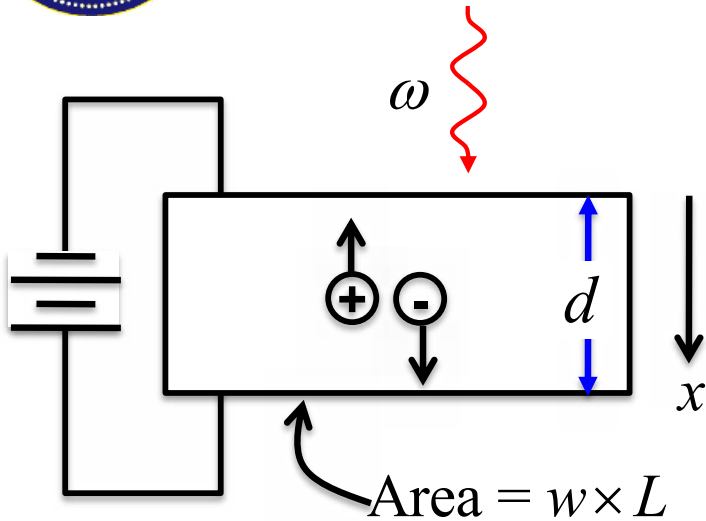
$$\text{RC-limited bandwidth: } f_{3dB} \approx \frac{1}{2\pi} \frac{d}{R \epsilon L w}$$

$$\text{Bandwidth-efficiency product: } f_{3dB} \times \eta \approx \left(\frac{1}{2\pi} \frac{d}{R \epsilon L w} \right) (\eta_i \Gamma \alpha L) = \frac{\eta_i \Gamma \alpha d}{2\pi R \epsilon w}$$

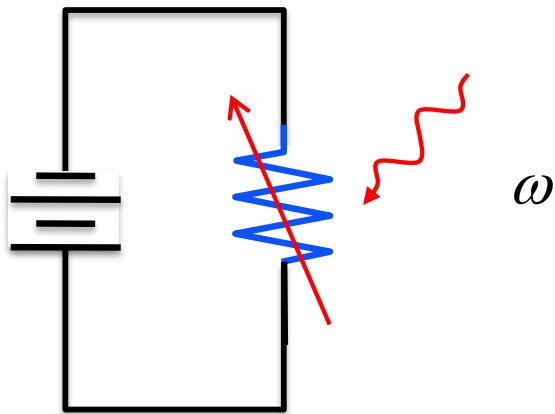
\Rightarrow In general, there is a bandwidth-efficiency trade-off



Photoconductors



Equivalent Circuit



Dark current:

$$J_0 = \sigma_0 E = (n_0 q \mu_n + p_0 q \mu_p) E$$

Light illumination generate electron-hole pairs, increasing the conductivity:

$$\frac{d\delta n}{dt} = G_0 - \frac{\delta n}{\tau_n}$$

Steady state: $d/dt \rightarrow 0$

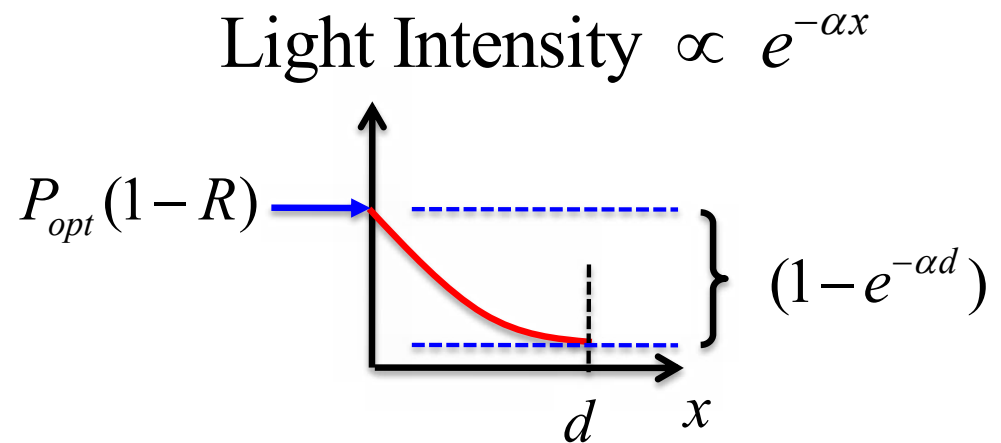
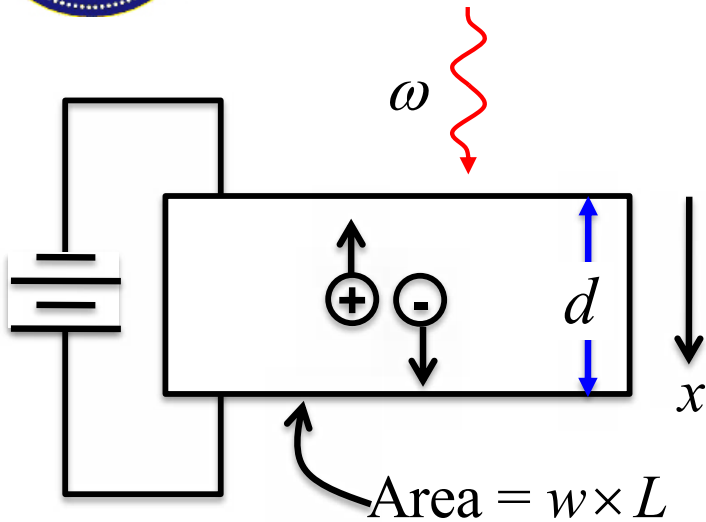
$$\delta n = G_0 \tau_n$$

$$\Delta J = \delta n \cdot q (\mu_n + \mu_p) E$$

Photoconductor requires both contacts to be Ohmic and the semiconductor doping type to be the same.



Photocurrent Generation Rate



$$G_0 = \eta \frac{P_{opt}}{\omega lwd} \frac{1}{lwd} : \text{photocurrent generation rate } \left[\frac{1}{\text{cm}^3 \text{s}} \right]$$

$$\eta = \eta_i (1 - R)(1 - e^{-\alpha d})$$

R : reflectivity of photoconductor surface

α : absorption coefficient

d : absorption length

$e^{-\alpha d}$: fraction of light remains after absorption length d



Photoconductive Gain

$$\Delta I = l w \Delta J = l w (G_0 \tau_n q) (\mu_n + \mu_p) E$$

$$\Delta I = l w \left(\eta \frac{P_{opt}}{\omega} \frac{1}{l w d} \tau_n \right) q (\mu_n + \mu_p) E$$

$$\Delta I \approx \eta \frac{P_{opt}}{\omega} \frac{1}{d} \tau_n q (\mu_n E) = \eta P_{opt} \frac{q}{\omega} \tau_n \frac{1}{d} v_n$$

$$\tau_t = \frac{d}{v_n} : \text{transit time}$$

$$\Delta I = \left(\eta P_{opt} \frac{q}{\omega} \right) \left(\frac{\tau_n}{\tau_t} \right)$$

Photocurrent

Photoconductive
Gain



Analogy to Current Gain in Bipolar Transistor

Current gain in bipolar transistor:

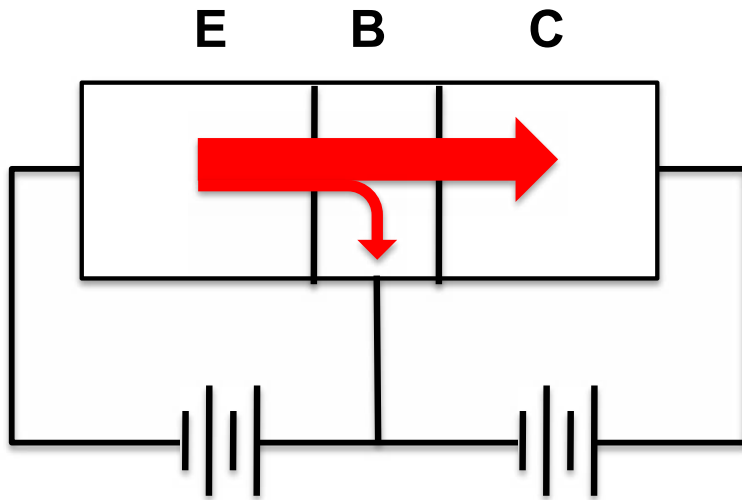
$$\beta = \frac{I_C}{I_B}$$

The current gain can also be expressed as

$$\beta = \frac{\tau_{rb}}{\tau_t}$$

τ_t : transit time

τ_{rb} : carrier recombination lifetime in the base





Frequency of Photoconductors

$$\frac{dN}{dt} = \eta \frac{P_{opt}}{\hbar\omega} \frac{1}{lwd} - \frac{N}{\tau_n}$$

Small signal response:

$$N = N_0 + N_1 e^{j\omega t}$$

$$j\omega N_1 = \eta \frac{P_1}{\hbar\omega} \frac{1}{lwd} - \frac{N_1}{\tau_n}$$

$$N_1 = \frac{\eta P_1}{\hbar\omega(lwd)} \frac{1}{j\omega + 1/\tau_n}$$

$$I_1 = J_1 lw = (N_1 q v_n) lw$$

$$\frac{I_1}{P_1} = \left(\frac{\eta q}{\hbar\omega} \right) \left(\frac{\tau_n}{\tau_t} \right) \frac{1}{j\omega\tau_n + 1}$$

$$= (\text{DC Quantum Efficiency}) \times (\text{Photoconductive Gain}) \times (\text{Normalized Frequency Response})$$

